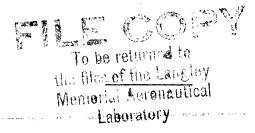TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 489

ON THE TAKE-OFF OF HEAVILY LOADED AIRPLANES

By Louis Breguet

From La Revue Scientifique, 1927, No. 12

Washington
November, 1928

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL MEMORANDUM NO. 489.

ON THE TAKE-OFF OF HEAVILY LOADED AIRPLANES.*

By, Louis Breguet.

The long nonstop flights recently carried out or undertaken, particularly the attempts to cross the Atlantic Ocean, have called attention to the difficulties encountered by airplanes in taking off with the large loads required for such flights.

This question, already studied elsewhere**, is again taken up here in order to investigate more thoroughly in the light of the knowledge acquired since then, the take-off conditions of airplanes equipped only with tractive propellers, and particularly the more difficult take-off of airplanes heavily loaded per unit of wing area (wing loading) or per unit of engine power (power loading).

Take-Off Phases

Let us consider the take-off run of an airplane and designate the weight of the airplane by P, the wing area by S, the speed by V, the density of the air by ρ, the tractive force of the propellers at full engine speed by T, the drag and lift coefficients by c_X and c_Z, and the coefficient of rolling friction on the ground by $\tan \psi$. This coefficient depends on the speed of the airplane. Particularly, in starting, it may

---

*"De l'essor des avions," from La Revue Scientifique, 1927, No.12.
**Louis Breguet, "De l'essor des aéroplanes," from L'Aérophile, April 1, 1908.

have a higher value than when taxying. For simplicity we shall adopt its mean value during the take-off.

The equation of the motion of the airplane on the ground is written:

$$\frac{P}{g} \frac{dV}{dt} = T - \tan \psi \left(P - \frac{\rho}{2} S c_z V^2\right) - \frac{\rho}{2} S c_x V^2. \qquad (1)$$

Evidently the first condition to be considered is that of starting the run. For this to be possible, it is necessary that

$$T_0 > P \tan\psi \qquad (2)$$

in which T_0 represents the propeller thrust when the airplane is standing still.

This condition being satisfied, the airplane starts on its ground run. The tail rises* and, in order to obtain the quickest possible take-off, the pilot must so adjust the angle of attack that the acceleration of the airplane will reach its maximum at any desired instant, i.e., for any value of V. The angle of attack thus defined is such that the quantity $\frac{\rho}{2} S (c_z \tan \psi - c_x)$ is constantly maximum. This angle of attack for maximum acceleration on the ground is constant. Obviously, it depends on $\tan \psi$, that is, on the nature of the field or of the take-off track. Let us designate it by i_i and let c_{x_i} and c_{z_i} be the corresponding coefficients.

---

*For the sake of simplicity I disregard the initial phase of taxying, which is of short duration for a well-balanced airplane and which lasts till the moment the tail leaves the ground. In other words, I assume that the airplane starts with its tail off the ground.

It is known that the fastest climb is that accomplished at the angle of attack i_2 of minimum power required for horizontal flight. The climbing rate of the airplane is then proportional to the difference between the maximum available power and the minimum power for horizontal flight, i.e., to the so-called "excess power." If this excess power is zero, or even very small, the aircraft is said to fly "tangent" or at its ceiling, that is, it can not get very far above the ground.

It is obvious that, in order to take off easily and with the greatest possible safety (especially with a "tangent" airplane), it is necessary for the pilot to adopt this angle of attack i_2 at the instant of taking off.

In order to take off under these conditions, the pilot must therefore maintain the angle of attack i_1 and run until the airplane has acquired the velocity of sustentation:

$$V_2 = \sqrt{\dfrac{P}{\dfrac{\rho}{2}\, S\, c_{z_2}}}$$

corresponding to the angle of attack i_2 . Raising the elevator at this instant will lead to a take-off at the angle of attack i_2 .

The angle of attack i_1, which gives the airplane its maximum acceleration on the ground, is found almost instinctively by a good pilot. In order to recognize the instant when the velocity attains the value V_2 (supposedly known in advance), the pilot can use an air-speed meter. At times, the pilot, feel-

ing the difficulty of leaving the ground at this speed, may be tempted to attain a speed considerably higher than V_2 before trying to take off. By virtue of its inertia, the airplane is then easily taken off, only to fall back again for lack of engine power. Such a leap may lead one to believe wrongly that the airplane is capable of taking off. It constitutes a grave fault of piloting, which may have the most unpleasant consequences both for the airplane and for the personnel.

The length L of the take-off run is evidently

$$L = \int_0^{V_2} V \, dt \tag{4}$$

Necessity for and Use of the Excess Power

Before calculating the run L, let us study the condition just mentioned, which is necessary for a successful take-off.

For taking off, it is necessary and sufficient for the tractive force T_e of the propeller at full throttle to exceed the drag of the airplane in horizontal flight at the speed V_2.

Let W represent the full-throttle power of the engine at the instant of taking off; η, the corresponding propeller efficiency; and, lastly, $\tan \varphi_2 = c_{x_2}/c_{z_2}$, the relative drag of the airplane at the angle of attack i_2 at which it should take the air. The above condition is represented by

$$T_e = \eta \frac{W}{V_2} \times 75 > P \tan \varphi_2 , \tag{5}$$

P, V_2, and W being expressed respectively, in kilograms, meters/second and horsepower.

The ratio:

$$\frac{T_e}{P \tan \varphi_2} = \frac{\eta W 75}{P \tan \varphi_2 \ V_2} = 18.75 \ \eta \ \frac{\sqrt{c_{z_2}}}{\tan \varphi_2} \ \frac{W}{P} \ \frac{1}{\sqrt{\frac{P}{S}}}$$

is the ratio of useful power with full throttle at take-off to the power strictly necessary in horizontal flight at the angle of attack i_2. We may therefore write "by definition"

$$1 + \epsilon = \frac{\eta \ W \ 75}{P \ \tan \varphi_2 \ V_2}, \tag{6}$$

in which ϵ denotes the relative excess power. Condition 5 may therefore be expressed in the form

$$\epsilon > 0 \tag{7}$$

This formula obviously states that, in order to take off, the excess power of the airplane must be positive at the take-off speed. This condition, though necessary, is not sufficient. We have already seen that the possibility of taking off also implies condition 2 in regard to starting from rest. Later on we shall find still another condition relative to the nature of the ground. Strictly speaking, the best angle of attack i_2 for climbing and which should be used in taking off, is the angle at which the relative excess power E is a maximum.

Disregarding variations in engine power and in propeller efficiency due to variations in the speed of these organs, the

angle of attack i_2 is the one for which the coefficient

$$\zeta = \frac{\tan \varphi}{\sqrt{c_Z}} = \frac{c_X}{c_Z^{3/2}}$$

(called "coefficient of power economy" of the airplane) is a minimum. In the case frequently occurring in practice, where the arc of the polar included between the angles of attack i_2 and i_m corresponding to the minimum relative drag $\tan \varphi_m$ is comparable to the arc of a parabola, the axis of which coincides with the axis of c_X. The lift coefficient c for the angle of attack i_2 is very nearly equal to the $\sqrt{3}$ times the lift coefficient c_{z_m} corresponding to the angle of attack i_m.

Since W and η vary with the angle of attack, the real angle i_2 of maximum relative excess power is in practice much closer to the angle i_m than the preceding theoretical angle. For c_{z_2} and $\tan \varphi_2$ we must therefore take values slightly greater than c_{z_m} and $\tan \varphi_m$, which correspond to the angle i_m.

The theoretical limiting load of an airplane is the one for which the relative excess power at the take-off speed is zero. This maximum load P_M can be calculated by the formula

$$P_M = \left(18.75 \, \eta \, \frac{W \sqrt{S \, c_{z_2}}}{\tan \varphi_2} \right)^{2/3} \qquad (8)$$

In order that the take-off may be made easily and safely, it is not only prudent but necessary for the load at take-off

to be kept considerably below the limiting value P_M indicated by the preceding formula.

The excess power at the take-off, thus reserved to the airplane, is useful from various viewpoints. In the first place, this excess power enables the airplane to rise easily above inequalities of the ground at the moment of taking off and in particular, to clear the obstacles ordinarily bordering aviation fields. It leaves, at the same time, a certain margin of safety in case of slight momentary engine trouble.at the moment of taking off. In the second place, this excess power makes it possible to begin the flight with only a fraction of the total power available. By reducing the output of the engines, we increase the reliability of their functioning. Lastly, it enables the pilot to climb quite rapidly from the very start, an indispensable requirement when his itinerary takes him over a mountainous region near the starting point.

For seaplanes and transoceanic flights, the initial ceiling of the aircraft may be somewhat lower. It is also conceivable that, in such a case, the excess power at the take-off might be considerably reduced. Experience has shown, however, that even in this case and for airplanes of average quality, one can hardly reduce this excess power below 15 to 20%.*

*L. Breguet, "Quelques reflexions sur l'aviation americaine et son avenir" (Some remarks on American aviation and its future), p.10 ff. (Roche d'Estrez, Paris, 1926). Laborious take-off of American seaplanes for the 1925 San Francisco-Honolulu flight.

In the flights made in 1926 with my airplane No. 19, "Record" type, I kept well above this limit, and the real excess power at the take-off was always about 60%. This very large excess power, on the other hand, enabled an easy take-off and climb in every case and in spite of the heavy load carried.

It seemed prudent not to go much below this figure, which insures a wide margin of safety, and in any case to consider the value of 25 to 30% as a practical minimum limit of the excess power for present landplanes designed for long flights and not obliged to climb very high at the start.

Evaluation of the Take-Off Run

Let us now evaluate the take-off run L, which represents the minimum ground run, on the assumption that the pilot keeps the airplane constantly under conditions of maximum acceleration. In order to carry out the calculation, we shall make a reasonable hypothesis regarding T in the equation of motion 1. Strictly speaking, T varies along the run. In fact, during this phase of the take-off, the rotational speed of the propeller generally increases slightly while V passes from 0 to V_2. This variation is such that T diminishes constantly from the value T_0 at the start to T_e at the instant of taking off. The ratio between T_0 and T_e depends on the propeller and on its adaptation to the engine. For propellers driven directly by the engine shaft and well adapted for the take-off, this ratio

varies between 1.2 and 1.6.

In order to calculate the run L in a simple way, we will consider T constant and equal to its mean value.

$$T = n\, T_e \tag{9}$$

on the run L, n being a number generally included between 1.1 and 1.3. The calculation of L must be divided into two distinct cases, according to the value of $\tan\psi$ with respect to $\tan\varphi_m$.

1. Let us first assume $\tan\psi < \tan\varphi_m$. The incidence i_1 of maximum acceleration is then less than i_m and consequently less than i_2. In order to show the effect of $\tan\psi$ on the value of i_1, it is advisable to relate c_{z_1} and $\tan\varphi$ to c_{z_m}, $\tan\varphi_m$ and $\tan\psi$. For this purpose, we may make the hypothesis (sufficiently exact for good airplanes) that the arc of the polar included between i_1 and i_m is comparable to the arc of a parabola having, for its axis, the axis of c_x and of the equation

$$c_x = a + b\, c^2\, z$$

By means of this hypothesis, we easily find that

$$
\begin{cases}
c_{z_1} = c_{z_m}\, \dfrac{\tan\psi}{\tan\varphi_m} \\[2ex]
\tan\psi_1 = \dfrac{\tan^2\varphi_m + \tan^2\psi}{2\,\tan\psi}
\end{cases}
\tag{10}
$$

On the other hand, the mean tractive force T, according to equations (9), (5), and (6), can be expressed by

$$T = n\, T_e = n\,(1 + \epsilon)\, P \tan \varphi_2. \qquad (11)$$

Consequently, the equation of motion (1) of the land run, at the angle of attack i_1 of maximum acceleration, may be written

$$\frac{P}{g}\frac{V dV}{dt} = P\,[n(1+\epsilon)\tan\varphi_2 - \tan\psi] - c_{z_1}(\tan\varphi_1 - \tan\psi)\frac{\rho}{2}\,S\,V^2 \qquad (12)$$

c_{z_1} and $\tan\varphi_1$ in this equation may be replaced by their values as functions of c_{z_m}, $\tan\varphi_m$, and $\tan\psi$ derived from equation (10). On integrating from 0 to V, we find by a simple calculation that the run L may be stated in the form

$$L = 3.75\,\frac{\tan\psi_m\,\frac{P}{S}}{c_{z_m}(\tan^2\psi_m - \tan^2\psi)}\,\times$$

$$\times \log\left[\frac{1}{1 - 0.5\,\dfrac{c_{z_m}}{c_{z_2}}\,\dfrac{\tan^2\varphi_m - \tan^2\psi}{\tan\psi\,[n(1+\epsilon)\tan\varphi_2 - \tan\psi]}}\right] \qquad (13)$$

In the limiting case, where $\tan\psi = \tan\varphi_m$, this relation reduces to

$$L = \frac{0.815\,\frac{P}{S}}{c_{z_2}\,[n(1+\epsilon)\tan\varphi_2 - \tan\psi_m]} \qquad (14)$$

2. Let us now assume that $\tan\psi > \tan\varphi_m$. The angle i_1 is then such that

$$\tan\psi > \tan\varphi_1 > \tan\varphi_m$$

The integration of equation (13) is somewhat modified. On carrying it through, we find that L takes the form

$$L = 3.75 \frac{\tan\varphi_m \frac{P}{S}}{c_{z_m}(\tan^2\varphi - \tan^2\varphi_m)} \times$$

$$\times \log\left[1 + 0.5 \frac{c_{z_m}(\tan^2\psi - \tan^2\varphi_m)}{c_{z_2}\tan\varphi_m [n(1+\epsilon)\tan\varphi_2 - \tan\psi]}\right]$$

(15)

showing in this case the possibility of a take-off which includes the mean coefficient $\tan\psi$ of the rolling friction of the airplane on the ground.

Influence of the Nature of the Ground on the Take-Off Run

According to formula (15) L becomes infinite, that is to say, the take-off becomes impossible as soon as $\tan\psi$ reaches the critical value

$$\tan\psi = n(1 + \epsilon)\tan\varphi_2 \qquad (16)$$

To the two previous conditions (2 and 7) of possible take-off, we must therefore add a third, namely,

$$\tan\psi < n(1 + \epsilon)\tan\varphi_2 \qquad (17)$$

The three conditions (2, 7 and 17) necessary for taking off, are then also sufficient.

Condition 17 interposes the nature of the ground. It is obvious that, theoretically, it may render the take-off impossible. We must consider, on the other hand, that for modern airplanes this can only occur for a relatively great mean coefficient $\tan\psi$ of rolling friction on the ground.

In fact, if an airplane of average qualities, 'for which

tan φ = 0.12 is taken as an example, the corresponding limit

of tan ψ defined by equation (16) (in which n is generally as

much as 1.1) would be at least equal to 1.1 $\times$ 0.12 = 0.132, if

the airplane under consideration has no excess power (ϵ = 0).

This limiting value of tan ψ must be multiplied by 1 + ϵ

whenever the excess power amounts to ϵ .

In practice, since some values of tan ψ appear entirely

exceptional, it may be said that the take-off of a fairly good

airplane, which has a normal excess power of at least

ϵ = 25 - 30% can not be rendered impossible, on any field which

is not exceedingly bad, by the mere influence of the nature of

the ground, i.e., by tan ψ . It is for this reason that little

attention is paid to the maintenance of our aviation fields,

which are generally rather poor. This does not mean, however,

that the nature of the ground (i.e., the value of tan ψ) has no

influence on the distance L traversed by the airplane before

taking off. Quite the reverse, the formulas (13) and (15) indi-

cate that L depends on tan ψ .

Application

In order to illustrate these theoretical considerations by

a numerical example, we shall consider a multi-engine biplane of

120 m². (1291.7 sq.ft.) wing area. We will assume that, at the

moment of taking off, it can produce 1200 HP., and that the cor-

responding efficiency of its propellers is 0.75, a very favorable

value for flying at the best climbing speed.

Lastly, let us assume that, for this speed, c_{z_2} and $\tan \varphi_2$ are respectively, 0.80 (or

$$K_{z_2} = \frac{\rho}{2} c_{z_2} = 0.05)$$

and 0.12, values which likewise appear very favorable for a multi-engine biplane, whose engines are not embedded in the wings.

Formula (6) enables us to calculate the relative excess power ϵ of this airplane according to the load carried. We thus find that, for the airplane loaded to 11,000 kg (24,250 lb.) at the take-off, the excess ϵ is 19%. This excess drops to 4.7%, if the airplane is loaded to 12,000 kg (26,455 lb.). Lastly, according to equation (8), it would fall to zero if the airplane were loaded to 12,400 kg (27,337 lb.).

Formula (8) also enables us to see that, in order for the airplane in question to be "tangent" at the take-off, with a total load of 13,000 kg (28,660 lb.) instead of 12,400 kg (27,337 lb.), it would suffice for all other conditions to remain the same; or for the propeller efficiency η to be increased from 0.75 to 0.805; or for the power W at the take-off speed to be raised from 1200 to 1290 HP.; or for the lift coefficient c_{z_2} to be increased from 0.8 to 0.915, (i.e., K_{z_2} from 0.05 to 0.0572); or, lastly, for the corresponding relative drag $\tan \varphi_2$ to be reduced from 0.12 to 0.11. These figures suffice to demonstrate the extreme difficulty that must be overcome in order

to increase the load of an airplane which is almost tangent, even by a small amount, without rendering the take-off absolutely impossible.

I have already indicated that, in order to take off satisfactorily under conditions of safety, there must be an excess power ϵ of at least 25-30% at the start. The influence of the nature of the ground on the take-off run is shown by applying the above-mentioned formulas to our example. Let us take $n = 1.2$ and for c_{z_m} and $\tan \varphi_m$ values somewhat lower than 0.8 and 0.12 already used for c_{z_2} and $\tan \varphi_2$. For example, let $c_{z_m} = 0.65$ and $\tan \varphi_m = 0.108$.

The coefficient of rolling friction $\tan \psi$ depends largely on the nature of the ground. On very good fields and for wheels mounted on ball bearings, it may be assumed, for example, that $\tan \psi = 0.03$. On uneven ground, such as exists on most aviation fields, we may assume quite high values of $\tan \psi$. In order to fix these ideas, let us take the successive values 0.03, 0.08, and 0.13.

By formulas (13) and (15) we find the following take-off runs:

(a) For 11,000 kg (24,250 lb.) load and excess power $\epsilon = 19.2\%$.

$\quad$ L = 775 m (2540 ft.) if $\tan \psi = 0.03$

$\quad$ L = 1115 " (3658 ") "   " = 0.08

$\quad$ L = 1830 " (6004 ") "   " = 0.13

(b)   For 12,000 kg (26,455 lb.) load and excess power ϵ = 4.7%.

   L = 3425 m (11,237 ft.) if tan ψ = 0.13

   L = 1485 " (4872 ") "   " = 0.08

   L = 1020 " (3346 ") "   " = 0.03

From this example it is obvious that tan ψ has a very noticea-
ble effect on the length of the ground run L.   This effect in-
creases, moreover, when the airplane load approaches the limiting
load P_M given by equation (8), the excess power of which re-
duces to zero at the moment of taking off.   The limiting value
which tan ψ would have to attain in our example in order to
render the take-off impossible, because of the nature of the
ground alone, would be, according to equation (16),

        (1.2 × 1.047 × .12) or 0.151 for
     an airplane loaded to 11,000 kg (24,250 lb.);

        (1.2 × 1.047 × .12) or 0.151 for
     an airplane loaded to 12,000 kg (26,455 lb.)

As already indicated, such values appear too large to be likely
to occur.

     For a well-kept flying field it seems reasonable to assume
that tan ψ always remains below a certain value, of the order,
for example, of 0.08 to 0.10.   From this viewpoint and admit-
ting this limitation of tan ψ,   it is obvious that, for the ex-
ample just considered, the airplane loaded to 11,000 kg (24,250
lb.) and having an excess power just sufficient for taking off
safely, should leave the ground after a run L of less than

1300 m (4265 ft.), however much below 0.10 $\tan \psi$ may be. Hence,
on an average field ($\tan \psi$ < 0.10), if a well-piloted airplane
equipped for long-distance flight does not take off in less than
1500 m (4921 ft.), it is because its power is insufficient for
the purpose and attempts to fly with that load should be discon-
tinued. In such a case it is certainly lack of sufficient engine
power and not the ground resistance which prevents the attain-
ment of the take-off speed V_2 . By prolonging this practically
maximum take-off distance in an effort to leave the ground, the
pilot would only invite a catastrophe, since the take-off, even
if possible, would be made with an insufficient margin of safety.
Consequently, if the take-off is unsuccessful at the indicated
maximum distance, the pilot should try at all costs to slow down
and stop.

It is evident, therefore, that, for a normal take-off on
average ground ($\tan \psi$ < 0.10), one should have a take-off track
of at least 2000 m (6562 ft.), thus reserving 500 m to stop the
airplane in case the latter fails to take off within 1500 m
(4921 ft.). If the take-off track is shorter than the above lim-
it, it is necessary, in order to shorten the run as much as possi-
ble, for the coefficient of rolling friction $\tan \psi$ to be as
small as possible. For a very heavily loaded or nearly "tangent"
airplane, the value of $\tan \psi$, as already indicated, greatly
affects the length of the take-off run. It is therefore very im-
portant, in order to facilitate the take-off of an airplane for

a long nonstop flight, to use a very smooth track (of cement, for example), or to take off from a track inclined toward the wind.

The foregoing leads to the conclusion that, in undertaking long nonstop flights for which one is compelled to use airplanes very heavily loaded, per unit of wing area and also per horse-power, it is necessary, under penalty of disaster at the start, to solve with approximate accuracy the difficult problem affecting the take-off of such airplanes.

Translation by
National Advisory Committee
for Aeronautics.